**Formal Properties of Green’s Functions**

still going...

**Differential equations obeyed by GF’s**

Let’s consider a free HO system with an external field added.



And let’s consider a few GF’s,

**Retarded GF**

And let’s consider GR(t,t´):



We can take the derivative w/r to time, and take advantage of the fact that the operator x will obey classical equations of motion, to arrive at:



The commutator between the two x’s is zero. And so we’re left with derivative of x. So let’s work that out,



which is what we expect. So now we have:



Now let’s take the derivative again,



The first term can be evaluated (note delta forces times to be equal) most easily by factoring out the time-evolution of the operators, evaluating the commuator, and then applying the time-evolution operator again. Of course since the commutator is a constant, this cancels out. And we get:



and now we need to evaluate the derivative of p…



And so we have:



Filling this into our GF equation,



and so we have:



where,



p stands for perturbation, by the way.

**Lesser GF**

Let’s do the lesser GF now.



So,



We already got that:



of course. So now we have:



Now let’s take the derivative again,



and now we need to evaluate the derivative of p…we found



Filling this into our GF equation,



and so we have:



where,



**Greater GF**

How about the greater GF?



So,



We already got that:



of course. So now we have:



Now let’s take the derivative again,



and now we need to evaluate the derivative of p…we found



Filling this into our GF equation,



and so we have:



where,



**Causal GF**

And we could do the causal GF, taking advantage of the algebraic relationship between the two: GC = GR + G<. Adding our two equations together:



we find:



where,



Interestingly, if F were position-independent, it wouldn’t show up in GR’s equation. But it would in GC’s, and G<’s, for instance. Well whatever. This lack of self-consistency is a reason why it’s not useful to try to develop a perturbative approach to the solution of the GF based off of its ODE.

**Anticausal GF**

Could work out the equation of motion for the anti-causal GF too. Let’s just use the property from above:



And then subtract the greater and retarded equations from each other:



or more succinctly,



where,

